

## A SOLAR MODEL WITH g-MODES

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### ABSTRACT

Good evidence is assembled showing that the Sun's core and surface vary on time scales from a month to a decade and that a number of scales are similar. The most plausible source for numerous long time scales and periodicities is long-lived global oscillations. This suggests g-modes (oscillations restored mainly by buoyancy) because they particularly affect the core and base of the convective envelope, which then indirectly modulates the surface. Also, standing g-modes have rotational properties that match many observed periodicities. But the standard solar model (SSM) has a static core and excites few if any g-modes, making new interior structures worth exploring. The model outlined here assumes two well mixed shells near 0.18 and 0.68  $R$  ( $R$  = solar radius) where sound speed data shows sharp deviations from the SSM. Mixing is sustained by flows driven by the oscillations. The shells form a cavity that excludes g-modes from their main damping region below 0.1  $R$ , assisting their net excitation and increasing their oscillation periods by at least a factor of two and probably much more. In terms of the solar luminosity  $L$ , the modes transport up through the cavity a power  $\sim 0.004 L$  as a lower limit and  $0.11 L$  as an upper limit. The modes dissipate energy in the outer shell and cool the inner shell, asymmetrically in each case, and this stimulates occasional convective events whose response time is typically 0.8 years longer near the inner shell. Such events cool the core and reduce neutrino flux while heating the envelope and increasing solar activity. This gives a physical basis for a well mixed Sun with low neutrino flux and basis for the observed anticorrelation and lag of neutrino flux behind surface activity.

*Subject headings:* Sun: interior—Sun: oscillations—Hydrodynamics—Sun: activity

## 1. INTRODUCTION

There is sustained interest in non-standard models of the solar interior—especially its core—even though the standard solar model (SSM) of Christensen-Dalsgaard, *et al.* (1996), Bahcall, Pinsonneault, & Basu (2001), and many others has been fitted to a number of observations. Guzik *et al.* (2001) review the subject and point out that alternatives to the SSM are motivated by difficulties with Lithium abundance and several aspects of the neutrino flux. Additional stimulus should come from evidence that neutrino generation varies with time (see §2.1), periodicities consistent with g-modes are detected (§2.3) even though g-modes are not expected in a SSM (Cox *et al.* 1991), the sound speed deviates from the SSM prediction (§3), and multi-decade variations such as the Maunder minimum and Spörer minimum suggest changes very deep in the Sun.

Non-standard models often introduce mixing. Ezer & Cameron (1968) first showed that arbitrarily mixing material into the core can improve agreement with the "low" neutrino flux and many have confirmed this in detail, recently Morel & Schatzman (1996). Mixing can also improve agreement with the p-modes (Gough & Kosovichev 1993) but, so far, not in the same model that fits massless neutrinos (Richard & Vauclair 1997; Watanabe & Shibahashi 2001). Schatzman (1969) first showed that the meridional circulation due to rotation causes turbulence and retards gravitational settling. More recently Zahn (1992) investigated its effects on angular momentum transfer. More rapid mixing happens due to gravity waves excited at the base of the convection zone that may reach the core (Schatzman 1993; Fritts, Vadas, & Andreassen 1998).

Cooling the core can also move the neutrino flux closer to agreement with observation, which Christensen-Dalsgaard (1992) showed by arbitrarily reducing the opacity in part of the core. Thin cool plumes (Cumming & Haxton 1996) descending into the core on million year time scales also decrease the ratio of  $^7\text{Be}$  to  $^8\text{B}$  neutrinos. Since the SSM has no core cooling or mixing, it predicts too large a neutrino flux but the possibility of neutrino oscillations (the MSW effect) gave new flexibility. Haxton (1998) compares the merits of MSW and mixing.

Unfortunately, modelers ignore a massive set of other observations—numerous regularities in solar behavior (§2), the Maunder minimum (Eddy 1976), and earlier such events (Stuiver & Quay 1980)—that have their most natural explanation in a core that can change rapidly and in a radiative interior pervaded by oscillation modes that obey the rotation law for standing g-modes (§2.3). To my knowledge, only Grandpierre (1996, 1999) squarely faced the solar activity observations and gave in some detail a model that places their ultimate source in the core with the neutrinos. He employs explosive events in very small portions ( $\sim 10$  km) of the core to create hot ( $> 10^8$  K) bubbles that rise, occasionally to the surface,

and presumably power surface activity.

A deep-seated source for solar variability is consistent with my own views (Wolff 1984), which require no rapid mass transfer up from the core or localized explosions. Instead, g-modes (Cowling 1941; Christensen-Dalsgaard & Berthomieu 1991) carry the energy upwards and also define the size ( $\sim 10$  Mm) of warm spots in the core where the modes are driven by locally nonlinear temperature oscillations. Herein I will outline a solar structure in which g-modes can thrive and thus explain a large number of periodicities seen at the surface and in the neutrino flux. In §2 the considerable evidence for regular fluctuations in solar observables is reviewed, including evidence for roughly 30 beat frequencies between g-modes. In §3, the observed sound speed curve is used to locate two layers where strong mixing bounds a cavity in which g-modes are trapped. In §4, key features of the model are described and §5 discusses limitations and tests of the model.

## 2. REGULAR TIME VARIABILITY IN THE SUN

It is well established that solar irradiance was 0.15% smaller near the 1986 minimum of the 11 year solar activity cycle than it was near the preceding or following maximum (Mecherikunnel 1996). The irradiance change is a true luminosity change since the p-mode frequencies also vary, implying a global effect (Frohlich 1993). To sustain a luminosity deficit over an interval of 4 or 5 years most likely points to a cause located below the convective envelope because much shorter overturning times are expected in the convective zone. The positive correlation between irradiance and solar activity on a multi-year scale coexists with an anticorrelation on a monthly scale as individual sunspots rotate into view (e. g., Hoyt, Eddy, and Hudson (1983)). We should not be surprised to learn that other quantities in the Sun suffer changes lasting for years and may correlate or anticorrelate depending on whether the time resolution is high or low.

### 2.1. Neutrino Flux

A rapidly varying neutrino production conflicts with the stable core of a SSM. Yet, if one ignores the challenge to popular models and recalls the great care taken by Ray Davis (1978) and colleagues in establishing error bars for each measurement from the Homestake neutrino detector, then a time-variable neutrino flux has been more likely than not since the 1980s. Sakurai (1979, 1980); Haubold & Gerth (1983, 1990); Raychaudhuri (1986b) and others felt encouraged to search for periodicities. They also speculated on new behavior in the core, as

did Wolff (1980). With more Homestake data available, Gavryusev and Gavryuseva (1994) could demonstrate high confidence levels (96% to 99.7%) for eight periodicities ranging from 11 months to 10 years. This indicates true time dependence in the production of neutrinos since the alternative (changes in the neutrino during its trip to Earth) cannot explain all these periodicities plus other variations cited below.

Sturrock & Scargle (2001) gave powerful evidence that the neutrino flux from the three Gallium detectors is bimodal. This calls to mind a long-noticed subset of Homestake runs consistent with zero neutrino flux separated by a small gap from the many high-count runs. Also, the Gallex data (Sturrock *et al.* 1999) and the Homestake data (Sturrock, Walther, & Wheatland 1997) show modulation by several periodicities comparable to solar rotation. Both are significant at the 99.9% level.

It is fairly likely that the secular trend of neutrino flux anticorrelates with trends in sunspot number and other activity on the surface, but failure to suppress short term fluctuations has confused the picture. Wilson (2000) cites many studies showing this anticorrelation and others that question their statistical significance. By first smoothing data over multi-month intervals, Davis, Cleveland, & Rowley (1987); Davis, Mann, and Wolfenstein (1989); Basu (1982); Sakurai (1980) and others found strong anticorrelations. Work disputing the anticorrelation (e. g., Walther (1997, 1999)) assumes there is zero phase lag between the secular extremes of neutrino flux and solar activity. This complaint is weakened (perhaps made invalid) by the nonzero lags reported by Raychaudhuri (1986a), Bahcall and Press (1991), and used by Oakley, *et al.* (1994) to find a highly significant anticorrelation with low latitude activity.

A definitive study should allow for the possibility that secular and short term trends correlate oppositely with surface observations and have different phase lags. The analysis of Bahcall and Press (1991) comes closest to this ideal. Their Figure 6 shows that the line they draw for declining neutrino flux happens about 0.8 yr after the rising portions of two 11 year solar activity cycles, confirming the  $\cong 1$  yr lag reported by Raychaudhuri (1986a). They reject using this fact because their opinion ("causality") would have the neutrino flux changing first. But I see their figure as good, uncomplicated evidence of a secular anticorrelation between sunspot number and a lagging neutrino flux. The fact that neutrinos lag will be compatible with our proposed solar model (see §5.1).

One more study of unreported statistical significance is worth mentioning because it shows large changes (factor of 2) in neutrino flux averaged over a good fraction of a year. Raychaudhuri (1996) assumed, as do some others, that a solar cycle has two peaks with different characteristics. Near these times (1990.0 and 1991.7) he inspected roughly 9 month segments of neutrino data as available and found that all three then-operating detectors

saw many more neutrinos near the second sunspot peak than the first. Increases were by the factors 2.0 (Homestake), 1.5 (Kamiokande), and 2.7 (SAGE). Chauhan, Pandey, & Dev (1999) tabulated this result again and speculated on how a neutrino magnetic moment several orders of magnitude larger than currently expected might cause rapid flux variations.

In summary, the rich variety of time dependences in neutrino data cited above—especially the periodicities  $\sim 1$  month to  $\sim 10^2$  months—will be more naturally explained by solar oscillation modes (§2.3) than by postulating numerous new properties for the neutrino and the medium they pass through.

## 2.2. Diameter

Ribes *et al.* (1991) reviewed studies of solar diameter. They show quasiperiodic behavior that anticorrelates with the 11 year solar cycle. How much of this is a true diameter change below the surface layers is unknown but real temporal changes on the Sun are being measured. Laclare (1983), Ribes *et al.* (1988), and Leister and Benevides-Soares (1990) found large peak to peak fluctuations of about 500 mas (milliarcseconds) in the semi-diameter recurring about every 950 days, which Wolff (1992) noticed was consistent with rotational properties of some g-modes. Under the g-mode interpretation, the 950 day tendency in diameter measurements is not continuous but will repeat at 35 year intervals. From its greatest strength in about 1978, the tendency should have declined to zero in about 1996 and should reach full strength again in 2013. So far, this expectation remains viable. Diameter fluctuations after 1988 were clearly smaller (typically 200 mas in the data which Laclare *et al.* (1996) reported). Most recently, Emilio *et al.* (2000) reported only 35 mas for two years centered on 1997.4, which happens to be near the minimum of the 35 year g-mode cycle.

Over the well known 11 year cycle, Laclare *et al.* (1996) measured a peak to peak diameter change of roughly 200 mas that anticorrelates with the solar activity cycle while Wittmann and Bianda (2000), measuring only at low latitudes, find about half that. Perhaps the difference is due to a simultaneous solar shape change as suggested by Kuhn *et al.* (2000). Two observations (Noel 1997; Ulrich and Bertello 1995) that contradict this anticorrelation seem to be overly impacted by solar activity since they show short term diameter fluctuations two to four times as large as the lower-noise results of Laclare *et al.* (1996) for the same epoch. Over longer intervals, some weak evidence indicates that diameter anticorrelates with the possible Gleissberg cycle (Gilliland 1981; Parkinson 1983) and with the singularly quiet Maunder minimum (Ribes *et al.* 1991).

The work of Davis, Mann, and Wolfenstein (1989) completes the correlation circle by

tying the diameter directly to long term trends in neutrino flux. The correlation is positive. Thus, the literature described in §2.1 and §2.2 supports the statement that, on a decadal time scale, diameter and neutrinos correlate with each other while both anticorrelate with the solar activity cycle, as first suggested by Delache *et al.* (1993). Observationally appropriate time lags among the three will be displayed in §5.1.

### 2.3. Beat Periods in Solar Activity, Diameter, and Neutrino Flux

In the  $1\frac{1}{2}$  centuries since Schwabe (1844) first documented a sunspot cycle and declared a possible period of “about 10 years”, many other periodicities have been suspected. Some have a proven high statistical significance; e. g., Rao (1973), Knight, Schatten, and Sturrock (1979), Lean and Brueckner (1989), Bai and Sturrock (1993), Ballester, Oliver, & Carbonell (2002). The reality of these periodicities has been questioned because they are not seen all the time. But intermittence is actually expected if one observes through a time dependent filter or if the data set is too short to resolve closely spaced periods. Consider a signal  $S(t)$  of five pure sine waves,

$$S = \sum_{i=1}^5 \sin(2\pi\nu_i t) \quad (1)$$

whose frequencies  $\nu_i$  are all comparable to solar rotation. A Fourier spectrum of 200 years of  $S(t)$  contains, of course, only the five frequencies (Figure 1a). Nonlinearities make the spectrum richer. The spectrum of  $S^2$  on figures 1b and 1c consists of second harmonics and ten beat frequencies ( $\nu_i - \nu_j$ ). A short data set cannot reliably demonstrate these beat frequencies as one can see in a Fourier spectrum of successive 20 year intervals of the  $S^2$  signal (figures 1d, e and f). When solar data shows such discordance from epoch to epoch (“intermittent periodicities”) people wrongly assume the periodicities are statistical, not real. Yet the signal underlying all parts of figure 1 is perfectly periodic and free of noise.

This illustrates the difficulty of detecting beats between the rotation rates of solar g-modes. Some beats require a century of data to resolve. The g-mode rotation rates provide raw signals similar to  $S$  and  $S^2$  below the convective envelope where most mode energy resides. But the modes become more detectible after nonlinearly exciting large convection cells in the envelope, which imposes those signals on observables at the surface. Most standing g-modes (the sum of two running modes of  $\pm m$ ) rotate at rates very close to each other as on figure 1a. The rates are especially simple (Wolff 1974) if tiny perturbations are available in the excitation region to lock the azimuthal states of a given  $\ell$  (the principal index of the spherical

harmonic  $Y_\ell^m$  describing the mode). Then asymptotic g-modes rotate at a rate

$$\nu_\ell = \nu_\infty \left[ 1 - \frac{1}{\ell(\ell+1)} \right] \quad (2)$$

which depends only on  $\ell$  and a rotation rate,  $\nu_\infty$ , for the Sun's radiative interior. This sequence and its beats are a signature of nonlinearly coupled g-modes in a slowly rotating star. Using 229 years of monthly sunspot records, Wolff (1983) identified almost two dozen beat periods from equation (2) between g-modes in the range,  $2 \leq \ell \leq 9$  as well as higher degrees (unresolved) with mean value  $\ell \cong 20$ . This was sufficient to convince me that standing g-modes were active inside the Sun and were modulating solar activity at the surface, presumably by stimulating occasional large scale convective events as the active longitudes of one set of modes ( $\ell$ ) slowly rotated past those of another ( $\ell'$ ).

Since equation (2) has been calibrated on the long sunspot record, including corrections  $< 1\%$  for cases  $\ell = 2$  and 3, its frequencies should be applied to modern data sets with no change. I did this (Wolff 1992) by identifying a g-mode origin for some conspicuous solar regularities reported by others; namely, five frequencies pervasive in solar activity, including the 155 day periodicity, and the multi-year fluctuation in the diameter data mentioned earlier. As a further illustration that the g-mode beat system influences the Sun, consider the work of Gavryusev *et al.* (1994). They analyzed the time dependence of the CERGA solar diameter data and listed the main periodicities, most of which have  $< 1\%$  chance of being accidental. Since they used a 30 day "month", their frequencies are multiplied by 1.014 and shown in the first column of table 1 in the proper SI unit. Every frequency in the diameter data lies close to a similar frequency found by Gavryusev and Gavryuseva (1994) in the Homestake neutrino measurements. Those are in the second column and never deviate by more than 1 nHz, which is reasonable for data sets covering 14 and 21 yr respectively. Since frequencies seen in two independent data sets have a greater chance of being real, those in the table are most suitable for comparing with the g-mode model. The last three columns list the  $\ell$  values of the g-modes, their beat frequency from Table 1 of Wolff (1983), and the deviation of the mean observed value from the beat. The agreement is excellent. The theoretical beat spectrum below 17 nHz is too dense to resolve with these data sets, and would produce erratic results like those on Figure 1.

### 3. SOUND SPEED DISCREPANCY AND WAVE LUMINOSITY

The sound speed,  $c$ , measured by helioseismology differs significantly from that expected in pre-existing solar interior models (Basu *et al.* 1996; Kosovichev *et al.* 1997; Basu *et al.* 1997). The fractional deviation has a narrow peak in the tachocline that will be modelled

by the positive half of a cosine curve

$$\frac{\delta c}{c} = A \cos[k_c(r - r_i)] \quad (3)$$

centered at  $r_i$  with amplitude  $A$  and wavenumber  $k_c = 2\pi(3w)^{-1}$  where  $w$  is full width at half height. Figure 2 shows this peak and another discrepancy of opposite sign located in the core. Both features ride on a broader discrepancy, drawn as a wide arc whose detailed shape will not be needed. Values used in equation (3) to plot the two marked features are printed on the figure and  $R$  is the solar radius. These six constants closely typify the last several figures of Basu *et al.* (2000). Since  $c^2$  is proportional to temperature divided by mean molecular mass  $\mu$ , each discrepancy can be interpreted as an excess of temperature, a deficit in  $\mu$  or both. Changing  $\mu$  is possible since the composition of solar models is apparently fairly flexible at the percent level. Brun, Turck-Chieze, & Zahn (1999) and Elliott and Gough (1999) saw no difficulty changing the helium mass fraction  $Y$  of a SSM by about 0.01 and then assuming mixing well below the present convection zone. This supplies hydrogen-rich fluid to the tachocline (making  $\mu$  smaller) and can remove a major portion of the discrepancy if the assumed mixing extends down a certain distance. Mixing in only the needed layers has also been proposed to explain the negative feature in the core (Richard & Vauclair 1997) or the abundance profile (Antia and Chitre 1997) but that mixing is *ad hoc*.

The present paper explores the other extreme: that the discrepancies are due to a perturbation  $T$  in the temperature  $T_0$  expected from the solar model,

$$T/T_0 = \delta c^2/c^2 = 2\delta c/c, \quad (4)$$

sustained by energy exchanged with waves (g-modes). Then the wave energy must be in equilibrium with radiative diffusion. The rate per unit volume of radiant energy gain is  $\dot{E} = -\nabla \cdot \mathbf{F}$  where  $\mathbf{F} = -\chi \nabla T$  is the radiative flux and  $\chi$  the thermal conductivity. Since  $r^2\chi$  in the divergence varies with  $r$  slowly compared to  $T$ ,

$$\dot{E} \approx \chi \frac{d^2 T}{dr^2} \quad (5)$$

is a useful (but rough) approximation over the width  $w$  of each feature. The total perturbed energy per unit volume is

$$E \approx E_0 \frac{T}{T_0} \quad (6)$$

Where  $E_0 = p_0/(\gamma - 1)$  is the thermal energy density. Equation (6) approximately includes work done against gravity in the vicinity of the two features. (If  $T$  had grown in zero-gravity at constant pressure, the right member would be multiplied by  $\gamma - 1$  and the equation would



be exact.) The radiative time constant  $E/\dot{E}$ , reduces to

$$\tau = -\frac{E_0}{\chi T_0 k_c^2} \quad (7)$$

with equations (3 & 4). When evaluated at the inner and outer features, respectively,  $\tau = 1.3 \times 10^6$  yr and 4600 yr.

Let  $V$  be the volume of a spherical shell of width  $w$  centered on  $r_1$  or  $r_2$ . Then the total radiation exchange rate is about  $V\dot{E}$ . This equals 0.11 L for the outer shell, where L is the solar luminosity. Waves must deposit this much energy to maintain a steady state (if the feature on figure 2 is due to T alone). Similarly, waves must remove about 0.02 L from the inner shell. The difference of 0.09 L would then represent power directly pumped into the g-modes by their nuclear excitation mechanism. For a zeroth order plausibility check let the waves carry 10% of the solar luminosity. Then the interior can remain consistent with the measured run of sound speed if temperature and mean molecular mass are each reduced by about  $2\frac{1}{2}\%$  between  $r_1$  and  $r_2$  compared with a SSM. The  $2\frac{1}{2}\%$  temperature change reduces the radiative flux by roughly 10% to allow for the wave luminosity. The reduction in  $\mu$  can be achieved by assuming a Hydrogen mass fraction,  $X$ , larger by 0.020 than in a SSM while keeping  $Z$  constant.

Smaller changes are quite possible since not all the sound speed discrepancy need be caused by a temperature error. But there is a lower limit to the g-mode luminosity since our g-modes would be the main energy source for the east-west flow that reverses about every 1.3 years in the tachocline (Howe, *et al.* 2000). I estimate the average power to reverse a laminar flow like this is  $\cong 0.002$  L and twice that after assuming equipartition with turbulence. This does not include power for the inevitable (in this model) flows at deeper, more dense levels. It seems safe to say that g-modes in this model have to transport at least 0.004 L across a large part of the radiative interior.

#### 4. A SOLAR STRUCTURE COMPATIBLE WITH g-MODES

There is precedent and plenty of evidence for seeking a nonstandard solar model. Extensive observations cited in §2 strongly suggest a Sun with many long periodicities, many g-modes, and a neutrino flux varying on monthly and decadal scales. This kind of evidence has been ignored by model builders for too long.

Helioseismology gives more evidence of an active core (Antia and Chitre 1997; Chaplin, *et al.* 1997; Gough & Kosovichev 1993) which has led some to compute models in which the core is arbitrarily mixed (Richard & Vauclair 1997; Brun, Turck-Chieze, & Morel 1998).

None of this is compatible with a SSM which is essentially static over the inner 98% of its mass. It is worth remembering that the p-mode probes of helioseismology offer the vast majority of their sensitivity in the convective envelope and thus have never supported solar models as securely at greater depths.

#### 4.1. The Model in Brief

Compared with a SSM, the structure proposed here has a much smaller buoyancy frequency,  $\nu_b = (Brunt - Vaisala)/2\pi$ , in the two shells where the sound speed error curve on Figure 2 has narrow features. The reduction is caused by mixing more rapid than the local radiative equilibrating time (Eq 7). The mixing is due to flows driven by g-mode dissipation in the tachocline and excitation in the core as explained in §4.3. The inner shell of low  $\nu_b$  excludes most g-mode energy from the sphere  $r < r_1$  where severe radiation losses would damp such modes in a SSM. It also makes oscillation periods longer than currently expected, partly explaining why no claimed detection of g-modes has been widely accepted.

Figure 3 is a cut through the Sun at a low latitude illustrating several elements of the model. A family of standing g-modes having the same value of  $\ell$  is excited in two small source regions near  $r_1$  where the modes sum near their innermost antinode to cause rms temperature fluctuations that are nonlinear. Once excited here, the g-mode family must dissipate small fractions of its energy throughout the Sun but especially in the tachocline near  $r_2$  where radiative diffusivity is a maximum. The dissipation near  $r_2$  on the figure is an asymmetrical heat source for the base of the convection zone, increasing the probability that a large scale convection cell or plume will erupt and modulate solar activity. Similarly, angular asymmetry in shell temperature at  $r_1$  leads to occasional overturnings. Since both upper and lower cells are driven by the same g-modes, the cells can modulate both surface activity and neutrinos on the same time scales (months and years) at which sources of several different families rotate into temporary overlap causing the larger convective events.

#### 4.2. Local Nonlinearity and Active Longitudes

Since the rotational beat frequencies discussed in §2.3 are a signature of g-modes nonlinearly coupled in the excitation region, their detection implies that significant temperature oscillations should occur somewhere in the Sun's core. I toyed with this idea long ago (Wolff 1980) and suggested that groups of g-modes would add constructively in a relatively small fraction of the shell,  $0.15R \pm 0.10R$ , to create locally nonlinear fluctuations while remaining

essentially linear over most of the Sun where their amplitudes add more randomly. The result was that, when the temperature fluctuation  $T > 0.05T_0$  in the shell, then the main nuclear reactions with their extreme temperature dependence provided an excitation that overwhelmed all conventional g-mode damping mechanisms since the latter grow with lower powers of the amplitude. This is intuitively obvious except for the size of the constant. Once there is net excitation, g-mode amplitudes run away. For an ultimate limit on their growth, I suggested displacements in the driving region that were too extreme to fully return during the second half of the oscillation period. Two more limits are mentioned in §5.2.

For a quarter century this picture has seemed reasonable to me, as has the likelihood that many of the azimuthal states of g-modes of a given  $\ell$  will overcome fractional differences  $\sim 10^{-5\pm 1}$  in their rotation rates to lock together in longitude in a way that maximizes release of energy from the nuclear term (Wolff 1974). The maximal solutions concentrate oscillatory power so that each family of g-modes, a "set( $\ell$ )", has a pair of active longitudes sloping with latitude. Figure 4 shows the sets for  $2 \leq \ell \leq 5$  in cases where each m-state has the same amplitude. These angular distributions are essentially identical to the 1974 solutions but one can notice minor improvements due to today's computing power and scan resolution.

Consistent with the above (that a nonlinear driving mechanism favors the concentration of power into small areas at large amplitude) one can assume that the radial eigenfunctions will adjust their signs to add in a similarly advantageous way. The innermost antinode is the likely place. It is near  $r_1$  because of the sharp drop in buoyancy frequency. (The lowest several radial harmonics are exceptions.) In this picture, the nonlinear temperature fluctuations for a set( $\ell$ ) occur in a small "source" region, concentrated in all three dimensions according to the size of  $\ell$  and the typical radial harmonic number  $n$  (Figures 3 & 4). Outside the source, the amplitudes in a set sum more randomly, produce few nonlinear excursions, and each individual g-mode behaves like a linear mode. Since the great majority of oscillatory energy lies outside the small source region, the rotation of g-mode sets should closely approximate the linear law (eq. [2]) and each pattern on Figure 4 should rotate at its unique rate  $\nu_\ell$  inside the Sun. The physical significance of the beat periods is now clear: As the prime longitudes of two sets rotate past each other, their sources largely overlap and cause nuclear burning to grow nonlinearly with their combined rms amplitude. This beat increases the angular asymmetry in temperature and, when large enough, large scale overturnings break out near  $r_1$  and  $r_2$  that modulate neutrino generation and solar activity in the outer convection zone. Detection of these beats in the long sunspot record (Wolff 1983) and in neutrino flux (§2.3) supports this scenario.

### 4.3. Long Oscillation Periods due to Mixing

Energy exchange with g-modes causes zonal flows to develop at low latitudes by a mechanism well known to meteorologists (Holton & Lindzen 1972) because angular momentum must be exchanged between g-mode and ambient fluid in proportion to how much energy is exchanged in the same location. Several groups suggested these flows for the Sun (Fritts, Vadas, & Andreassen 1998; Mayr, Wolff, and Hartle 2001; Kim & MacGregor 2001) and one such flow has been detected (Howe, *et al.* 2000) near  $r_2$  as discussed in §3. Zonal flow in a rotating star balances latitudinal momentum by driving a meridional circulation, which then perturbs the vertical temperature gradient closer to an adiabat. Guided by Fig. 2, I assume that vertical mixing is strongest near  $r_1$  and  $r_2$  and that it is strong enough to cause a severe local reduction in the buoyancy frequency. The dashed curve on Fig. 5a is a gross picture what the new  $\nu_b$  might look like. It is simply the product of  $\nu_b$  from a SSM (solid curve) and the factor  $f$  on Fig. 5b, which was constructed from the error function so that  $f$  makes 90% of its transition between 0 and 1 over the distances  $w$  already defined by the sound speed error curve. Incidentally, mixing that peaks near  $r_1$  will "appreciably reduce" (Morel & Schatzman 1996) the discrepancy between observed and theoretical neutrino fluxes.

The dashed curve will be used to estimate a lower limit for g-mode oscillation periods. For simplicity, this curve remains zero to the origin although mixing that deep is not essential. In fact, Richard & Vauclair (1997) find a conflict with helioseismology if mixing extends below about 0.12 R unless the diffusion constant is  $< 10^{-2} \text{ m}^2/\text{s}$ . The conflict might also be avoided by wave cooling (§4.4). In any case, an unmixed sphere well below  $r_1$  would barely affect the oscillation periods of the g-modes of interest, which have the great majority of their energy above  $r_1$ .

Fig. 6a shows the radial dependence of a typical g-mode in a SSM (solid curve) and in our proposed model (dashed), computed from the linear differential equations. The curves show the usually dominant angular component of motion for the case,  $(\ell, n) = (5, 5)$ . The oscillation frequency in the SSM, 0.283 mHz, is cut in half by the new  $\nu_b$  to 0.153 mHz. Further reduction will occur when lesser mixing at intermediate depths can be estimated in a complete solar structure integration. Then the buoyancy frequency of the SSM will be reduced everywhere, as in the wholly arbitrary dotted curve drawn on Figure 5a which will cut oscillation frequencies by a factor of seven. In summary, g-mode oscillation *periods* in this model are at least twice (and probably a lot more) than observers have been searching for, explaining why no period has been detected "convincingly" (Palle 1991) even though the rotational beats are seen (§2.3).

#### 4.4. Damping and Wave Cooling

The main g-mode damping in a SSM is radiative diffusion in the core,  $K\nabla^2 T$ , where  $K$  is the radiative diffusivity plotted on Figure 6b. It follows that radiative damping varies as  $KT/r^2$  since g-mode wavelengths are  $\propto r$ . The amplitude of  $T$  increases at successive antinodes closer to the core, so  $K/r^2$  (dotted curve) is a lower limit to relative damping as one moves down toward the innermost antinodes. One sees that the heavy damping suffered by most g-modes in a SSM for  $r < 0.1R$  does not affect modes in the new model because they barely penetrate this region.

When g-modes are excited in the source regions of Figure 3, they must receive heat there at maximum compression. This is explicit in a general energy equation for quasi-linear modes (Cox (1980), Chap.5).

$$\frac{d}{dt}\left(\frac{\delta T}{T_0}\right) - (\Gamma_3 - 1)\frac{d}{dt}\left(\frac{\delta \rho}{\rho_0}\right) = \frac{1}{c_v T_0}\delta\left(\epsilon - \frac{\nabla \cdot F}{\rho}\right), \quad (8)$$

which shows that when the nonadiabatic right member is positive, the temperature is still increasing when the density rate of change is zero. In this equation,  $\epsilon$  is the nuclear energy generation rate,  $F$  the radiative flux,  $c_v$  the specific heat per unit mass, and  $\delta$  indicates a Lagrangian variation. This temporal phase shift of  $T$  must exist for each individual g-mode being excited and persist over the entire shell  $r \cong r_1$  because radial and angular behaviors are independent for linear modes. But, by assumption, nuclear pumping is significant only in the source region where a whole family of g-modes adds constructively to cause a large fluctuation. This leaves most of the shell at  $r_1$  in the condition:  $\epsilon$  negligible, positive phase shift at maximum compression. By equation (8), only the radiative flux is left to supply the energy which the wave is absorbing here. Thus, each g-mode is acting like a refrigerator on most of the shell, cooling it. This mechanism should be considered in a detailed model as a way to explain the cool shell at  $0.18R$  apparent in the sound speed. It will be more effective at large  $\ell$  for which almost all the shell is refrigerated.

### 5. DISCUSSION

#### 5.1. Leads and Lags

Observations and reasonable inference from the model give preliminary information on chronology. Two sets of g-modes suffice for the first part of this discussion. As each family performs its rotation according to equation (2), there will be a time when the source of set( $\ell$ ) has maximum overlap with the source of set( $\ell'$ ). This situation (a "beat") maximizes the

rms temperature fluctuations in the volume common to both sources. A simultaneous rise in neutrino flux has to occur since the volume is in the nuclear burning core near  $r_1$ . The angular distribution of the overlapping is independent of  $r$  by the properties of linear modes, so these angular locations experience larger temperature fluctuations at all  $r$  and therefore larger radiative dissipation. Loses are particularly strong near  $r_2$  where radiative diffusivity peaks, as mentioned earlier. Since a beat maximizes angular differences in temperature, especially near the above two shells, thermally driven overturnings are stronger and more probable near those shells. When an overturning or plume erupts, it has to occur with some lag whose size is controlled by the strength of the beat and consequent response time of the fluid. Deep in the convective envelope, lags of many months are plausible before surface brightness and solar activity achieve their full increase. From the refrigerated shell near  $r_1$ , descending fluid will cool the core and reduce neutrino flux, again with some lag.

It is very common for two sets to overlap, causing minor short term changes. Rarely, a larger number of particularly strong sets will slowly reach a major degree of overlapping marking the culmination of a long term trend. Figure 7 places in order some events that follow each type of beat. The time when a long term trend ( $\sim 10$  yr) reaches an extreme is indicated by a spike of height  $\pm 1$  while smaller spikes indicate short term fluctuations ( $\sim$  months). Drawn symbolically at  $t = 0$  is a maximum of the international sunspot number  $R_I$  and a minimum of short term solar irradiance showing their known anticorrelation with zero lag. The spike for long term irradiance shows its known positive correlation with  $R_I$  but whether a lead or lag is undetermined although it will be less than 1 yr.

At right is the neutrino flux minimum, which lags the 11 year sunspot maximum by 0.8 yr as discussed in §2.1. Numerous papers establish the signs (correlation or anticorrelation) of spikes on Figure 7 but my reading of plots in two papers is solely responsible for the estimated 0.3 yr lead of solar diameter (Laclare, Delmas, and Irbah 1999) and 0.1 yr lag of  $R_I$  (Jimenez-Reyes, *et al.* 1998) from the long term maximum of p-mode frequencies. At left is the presumed cause of it all - the g-mode beat—and a simultaneous rise in short term neutrino production. Observations do not establish their position on the time axis but the virial theorem and rapid sound speed should place them nearly simultaneous with the minimum in solar diameter.

The main physical conclusion for long term trends is that convective response times in the core measure about 0.8 years longer than those in the deep convective envelope. If responses in the envelope are about 0.4 yr as drawn on Figure 7, then it takes 1.2 yr after a beat to fully lower the neutrino flux by means of cool sinking fluid. Some of this time may be consumed by nuclear populations equilibrating with a new temperature. Future correlation studies should allow for nonzero lags and focus separately on short and long term behavior.

## 5.2. Limitations and Tests of the Model

This paper can supply only an outline of an observationally adequate solar interior because of many uncertainties that call for new observations, new inversions of helioseismology data, and very inclusive numerical integrations:

- 1) What are the g-mode amplitudes?
- 2) What limits their growth?
- 3) What fraction of the luminosity do they transport?
- 4) If the  $^1\text{H}$  and  $^4\text{He}$  fractions are constant from surface to center, how many other components are mixed fast enough to be uniformly distributed?
- 5) How severely is the buoyancy frequency reduced over the bulk of the radiative interior?
- 6) What is the precise location and effective width of the rapid mixing near  $r_1$ ?
- 7) How far from spherical symmetry is the temperature in the shells at  $r_1$  and  $r_2$ ?

Several tests are feasible right now but it will probably take years to gradually incorporate all features and adjust parameters incrementally, as we've done for decades with the SSM. Answering questions 1 and 5 requires searching for g-modes in a range of longer periods and measuring the amplitude and oscillation period associated with each angular harmonic. Once the surface amplitude of a particular mode is measured, a fair estimate can be made of its amplitude at the base of the convection zone and the energy deposited there. This sets a lower limit to wave luminosity (question 3). It will be harder to estimate from g-mode amplitudes the additional luminosity that goes into east-west flows but soon helioseismology may directly measure these flow fields. The thermal asymmetry (question 7) should not be a difficult problem for helioseismology in the outer shell where the sensitivity is high. As data accumulates, similar information on the inner shell will become more reliable, shedding light on both questions 7 and 6.

The remaining questions (2 and 4) call for theoretical work. To answer item 2, requires solving fully nonlinear motion in the source. In addition to calculating how much of the fluid fails to return after a large displacement as mentioned in §4.2, two other possible limits should be investigated. The first concerns local fuel exhaustion. If any of the most effective fuels for exciting oscillations are rare, they may become depleted in the source region. Then, the burn rate and g-mode amplitudes will have to come into balance with the rate at which new fuel is sampled as the source drifts slowly in longitude through the solar fluid. Unfortunately, the

caloric value of the fresh fuel will be variable, depending on what other sources have recently sampled that fluid. A second possible limit on the amplitudes concerns the tachocline and convection zone where a small buoyancy frequency favors turbulence. Here, at the same latitude and longitude as the source, strong turbulence and mean flows will develop near  $r_2$  and extract considerable energy from the modes that is hard to evaluate.

Finally, any wave-driven mixing sufficient to drastically reduce the buoyancy frequency in the two shells must take place on time scales comparable to or shorter than a radiative relaxation time ( $\sim 10^5$  to  $10^6$  yr). But the dotted curve on Figure 6 shows that wave dissipation never drops more than a factor of ten below its value in the tachocline. This indicates that mixing everywhere between  $r_1$  and  $r_2$  will be rapid compared to the evolutionary times ( $\sim 10^9$  yr) on which the  $^1\text{H}$  and  $^4\text{He}$  populations change. Therefore, these components will be well mixed in this model above  $r_1$  and for an unknown distance below. But the population of some other nuclear species will depend strongly on the actual size of the vertical mixing rate and will be hard to calculate with certainty. The answer to question 4 may not come soon.

## 6. SUMMARY

The key features were identified for a solar structure suitable for sustaining many g-modes in order to accommodate numerous reports of rapid variations in neutrino flux, regular solar behavior on various long time scales, and many actual periodicities attributable to the rotation law for standing asymptotic g-modes. A model is no more acceptable if it cannot accommodate these many variations (§2) than if it gets the neutrino flux or internal sound speed wrong. The model proposed herein is only an outline because of numerous questions listed above but it may serve to guide future studies in productive directions. Magnetic fields played no role herein.

Some elements of the model are consequences of having significant power in g-modes nonlinearly coupled in small source regions (Figs. 3 & 4), as Wolff (1974) proposed. There will be an inner shell where the buoyancy frequency plunges, forcing the innermost antinodes to locate near the same radial distance where sources can periodically overlap and reinforce each other through highly nonlinear nuclear burning. There will be significant zonal flows and associated meridional circulations that mix the Sun near and above  $r_1$  in a time short compared to an evolutionary time scale (§5.2). Quasi-periodic convective events will break out above and below the cavity containing the g-modes due to enhanced angular asymmetries when many modes or the strongest modes overlap.



The shells bounding the cavity were centered at  $r_1 = 0.18 R$  and  $r_2 = 0.675 R$  where sound speed observations deviate most sharply (§3) from predictions of the SSM. Sets of g-modes pick up energy near  $r_1$  from nuclear burning in small source volumes and also from a refrigeration process (§4.4) that cools most of the shell. This cool shell can be seen in the sound speed data, which also shows a heated shell at  $r_2$  where g-modes will lose the most energy due to a maximum in radiative diffusivity (Fig. 6).

Estimates were made in §3 that g-modes transport between 0.4% and 11% of the solar luminosity across parts of their cavity. If this model is basically correct then: 1) g-mode oscillation periods are longer (§4.3) than observers have been searching for and probably much longer, 2) neutrino generation rates will vary on time scales as short as months and show more and more of the same periodicities as surface activity and diameter (Table 1) as data accumulates, 3) there should be at least one strong east-west flow near  $r_1$  where g-modes extract the most energy from the core.

By not ignoring the long running story of periodicities (§2.3) and long term trends that solar modelers have not heeded, by considering that g-modes might play an important role in the Sun's structure even though damped in a standard solar model, we induced a model that gives us a bonus that  $^1\text{H}$  and  $^4\text{He}$  are well mixed inside most of the Sun (§5.2), which is a known solution to the original neutrino flux problem, which models predicted too high.

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Table 1. NEW FREQUENCIES COMMON TO OBSERVATIONS & g-MODES

Diameter	Neut. Flux	<u>g-Mode Beats</u>		O-C
(nHz)	(nHz)	$\ell, \ell'$	Freq. (nHz)	(nHz)
18.9	17.9	5,10	18.5	-0.1
24.7	23.9	4, 7	24.5	-0.2
28.8	28.6	4, 9	29.6	-0.9
33.6	33.7	3, 5	32.8	0.8
37.1	37.3	4, G	36.3	0.9
40.6	39.6	3, 6	40.0	0.1
46.5	47.5	3, 8	47.6	-0.6





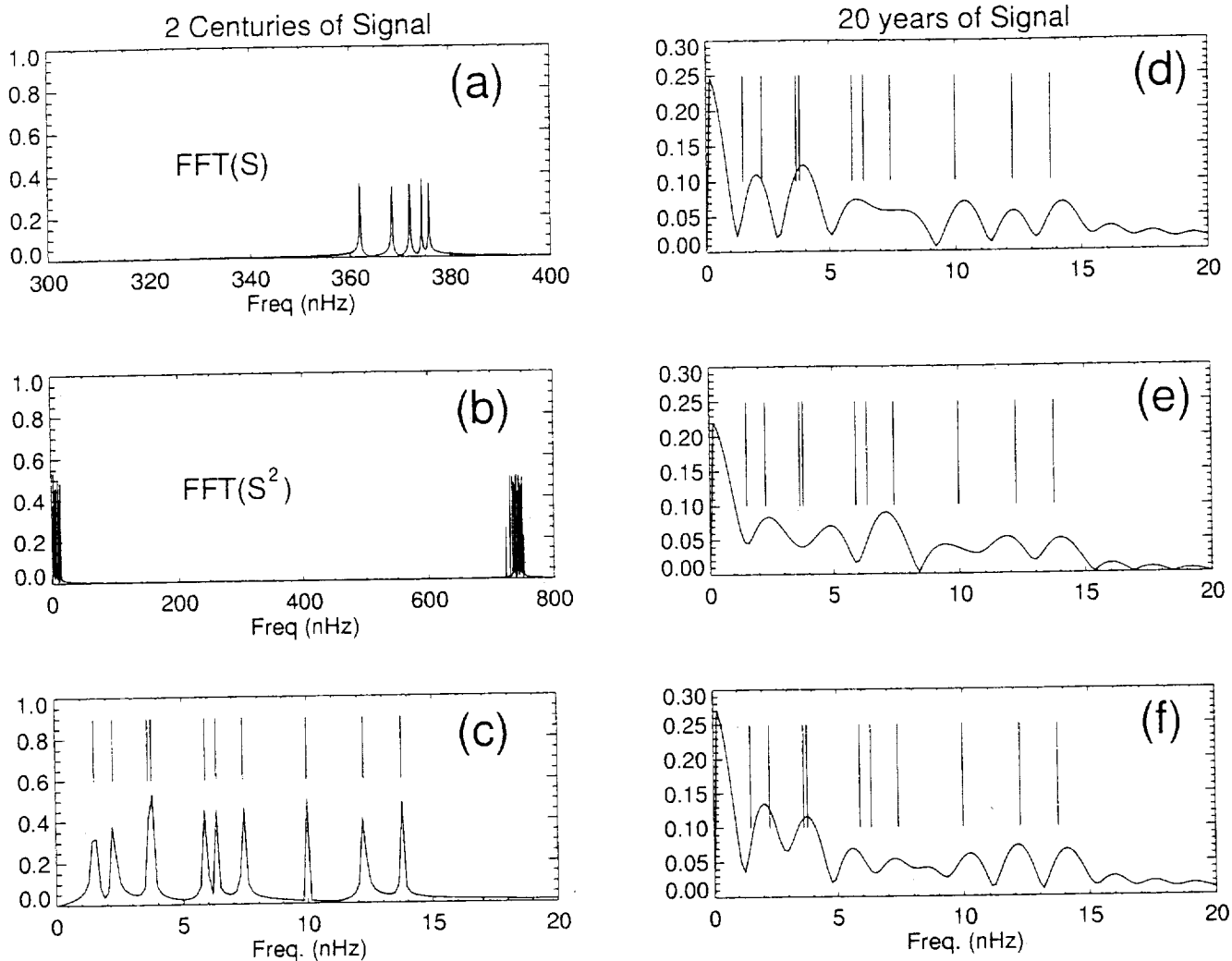


Fig. 1 Effect of nonlinearity and data length on a simple signal  $S$  whose 5 frequencies are all comparable to the solar rotation rate. *Left column:* Using two centuries of the signal, (a) the Fourier spectrum of  $S$  clearly shows the five frequencies but (b, c) the spectrum of  $S^2$  shows only the first harmonics and ten beat frequencies. *Right column:* Using successive 20 year segments of  $S^2$  to produce a spectrum (d, e, f) gives erratic agreement with the true beat frequencies (vertical lines). Beats near 3.5 and 6 nHz are not detected in the middle 20 years giving the impression that they are intermittent. Even the beat at 10 nHz, well separated from the others, seems to vary by  $\pm 1$  nHz from epoch to epoch.



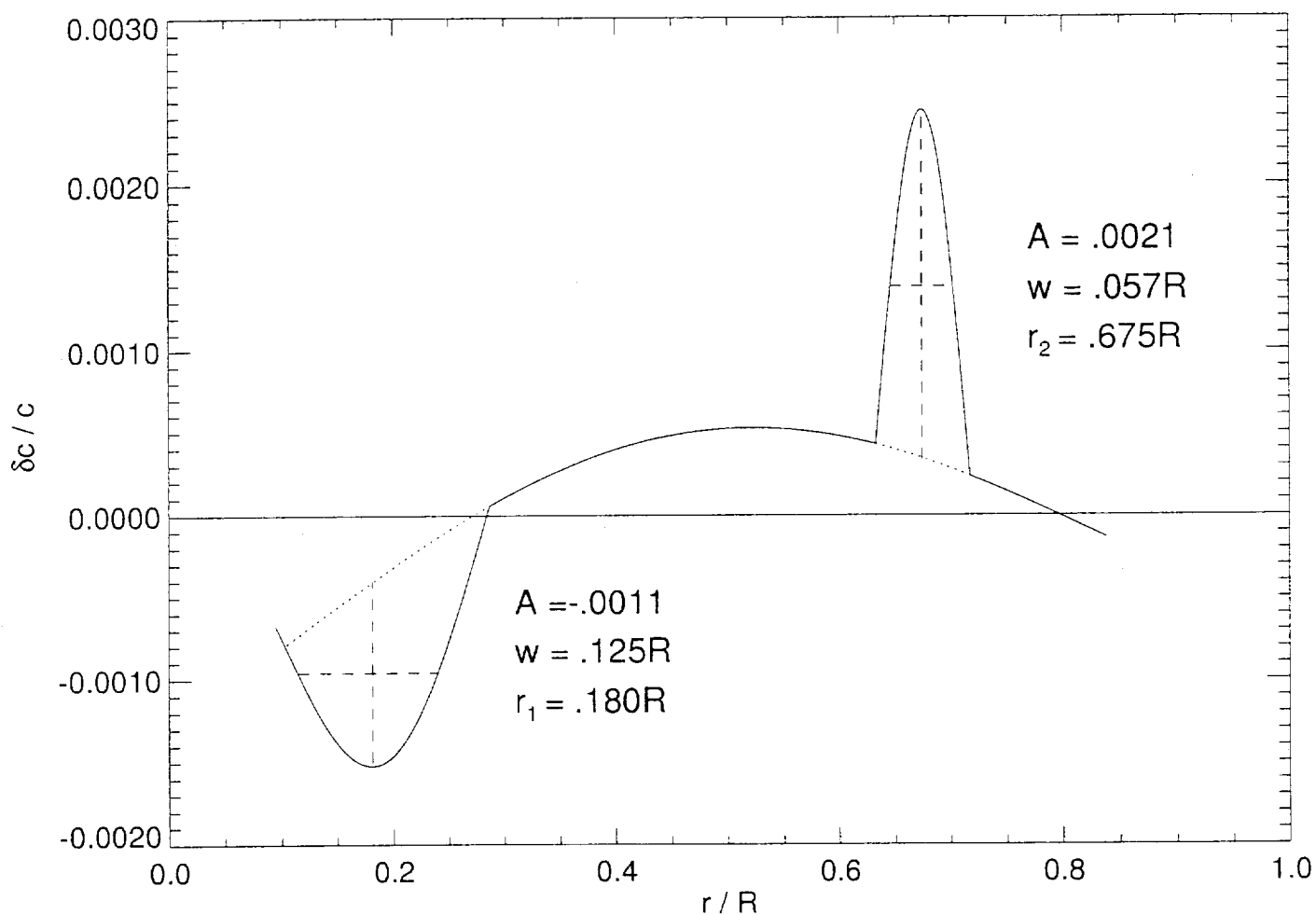


Fig. 2 The observed fractional deviation  $\delta c/c$  of the sound speed from a standard solar model is approximated here by two half-cosine curves resting on a broad arc. The two narrow features are defined by equation (3) with constants for amplitude, width at half height, and radial location printed on the figure. All six constants fit the analysis of Basu, et al. (2000).



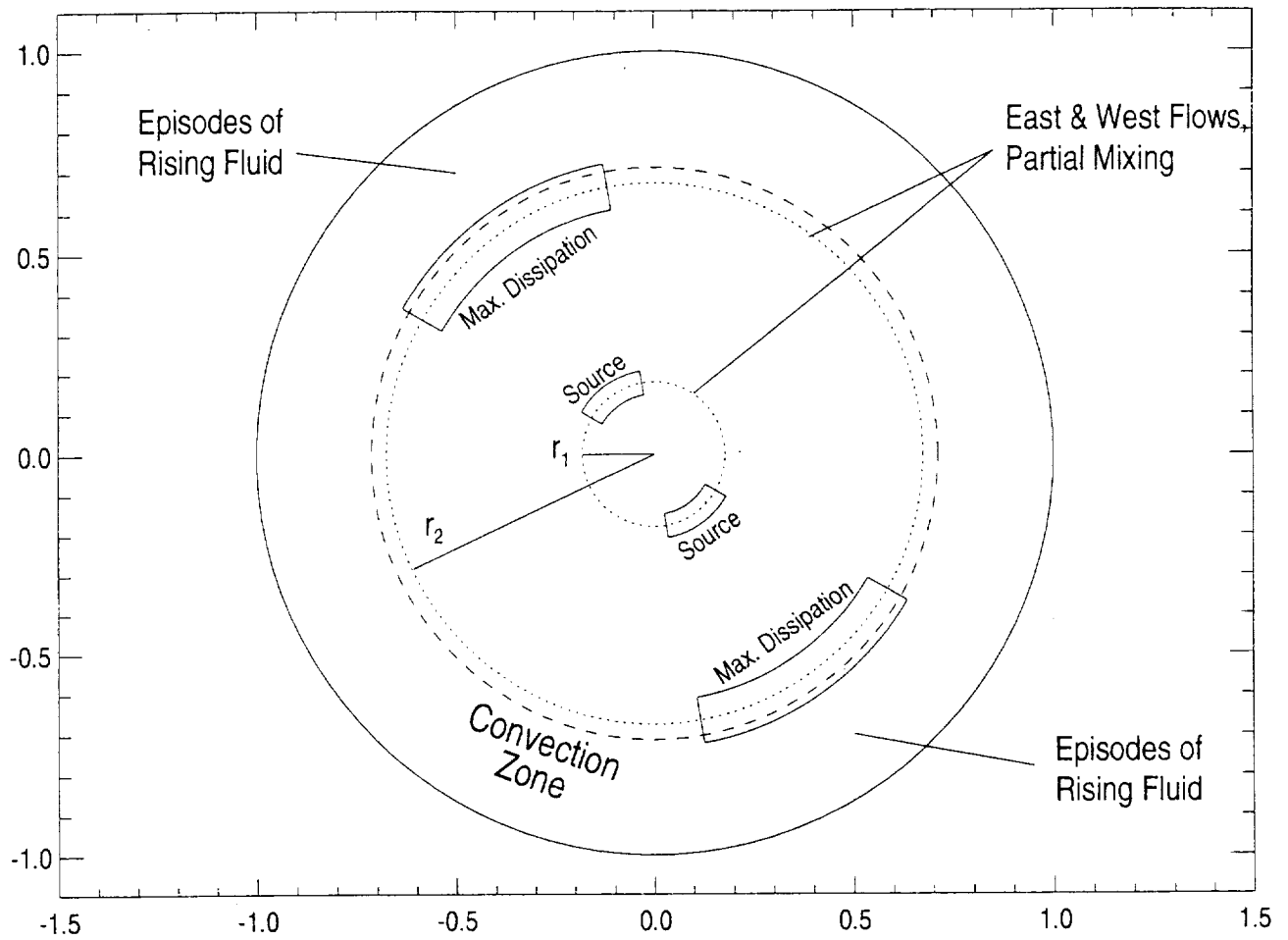


Fig. 3 The low-latitude behavior of the g-mode family for  $L = 3$  in the proposed solar interior. Nonlinear temperature fluctuations in the source region excite the modes, which deposit energy most heavily near the base of the convection zone. Zonal flows, especially in these shells, are driven by the g-modes and cause vertical mixing due to the Coriolis force.



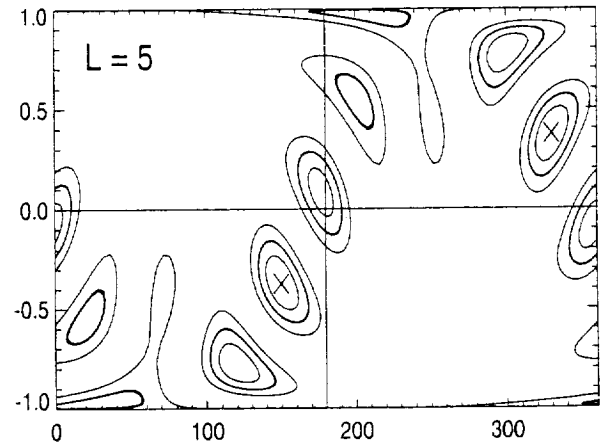
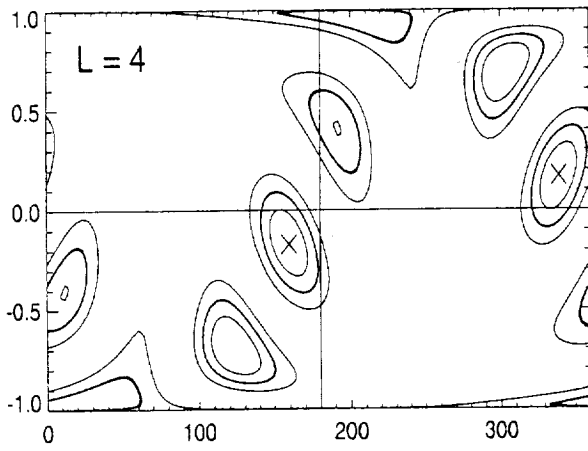
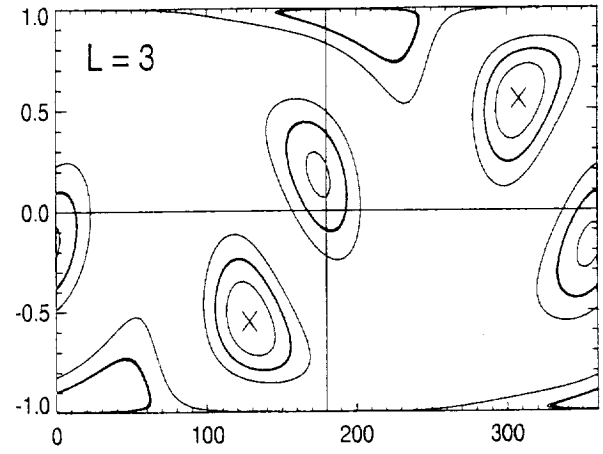
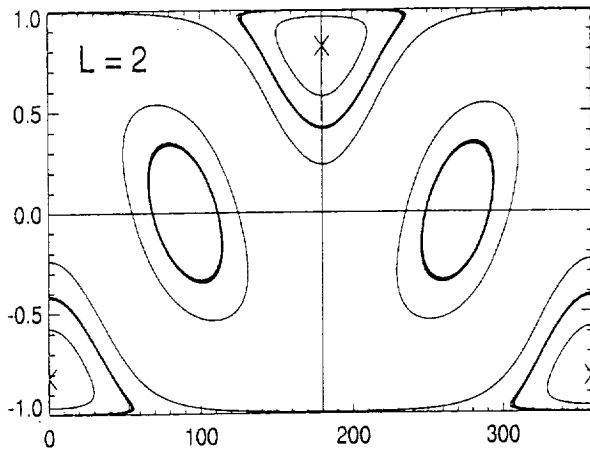


Fig. 4 Angular distribution of oscillator power expected for standing g-modes. Each pattern maximizes the effect of a strongly nonlinear mechanism such as nuclear burning. The patterns rotate at unique rates (Eq. [2]) and each is the sum of all azimuthal states,  $m$ , for the same value of  $L$ .





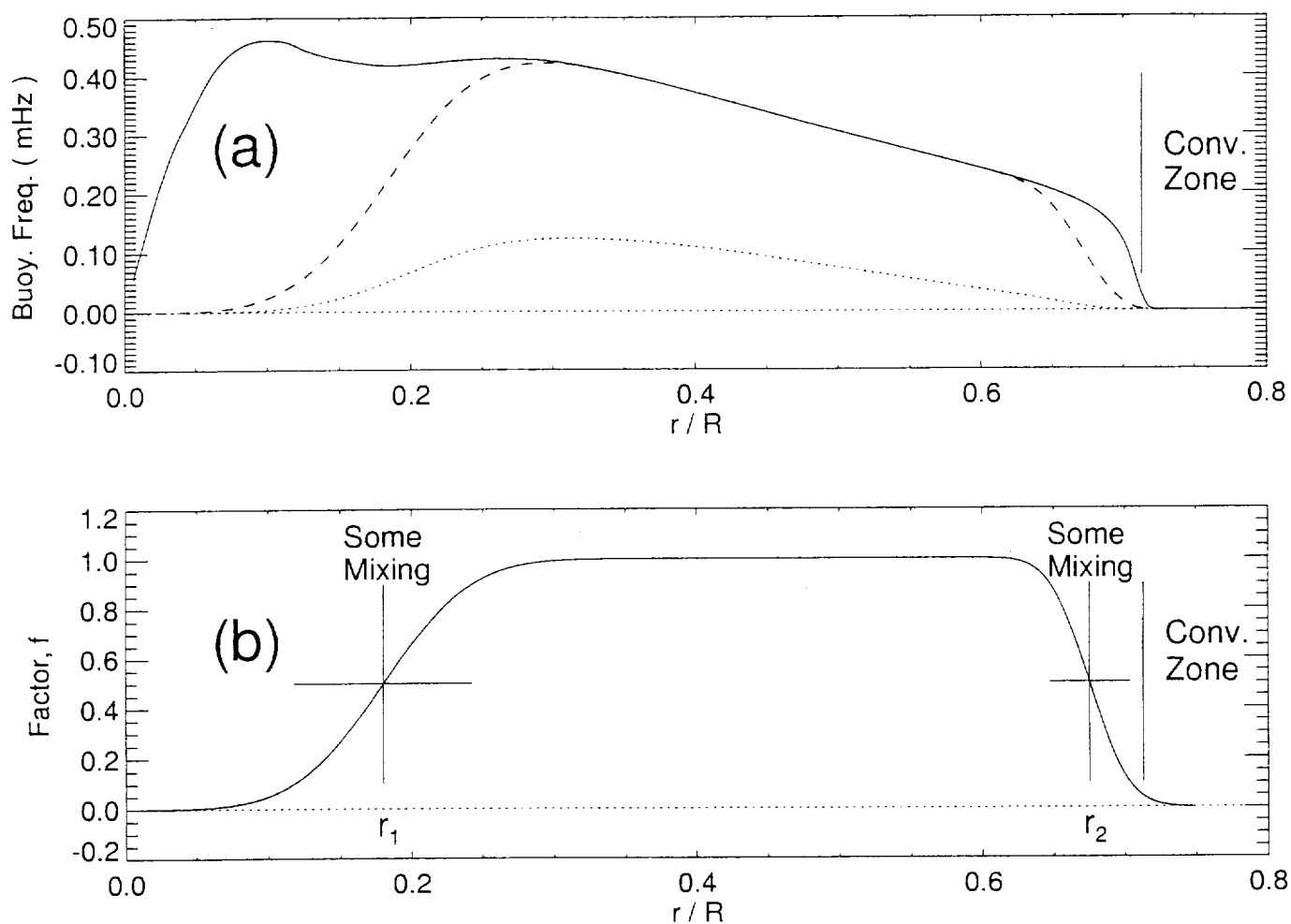


Fig. 5 a) The radial dependence of buoyancy frequency in a standard solar model (solid) and the proposed model (dashed), which includes vertical mixing that is strongest near  $r_1$  and  $r_2$  where discrepancies in sound speed are observed. The dotted curve merely indicates that further reduction is expected when slow mixing throughout the Sun is included. b) The factor by which the standard buoyancy curve was multiplied to get the dashed curve.



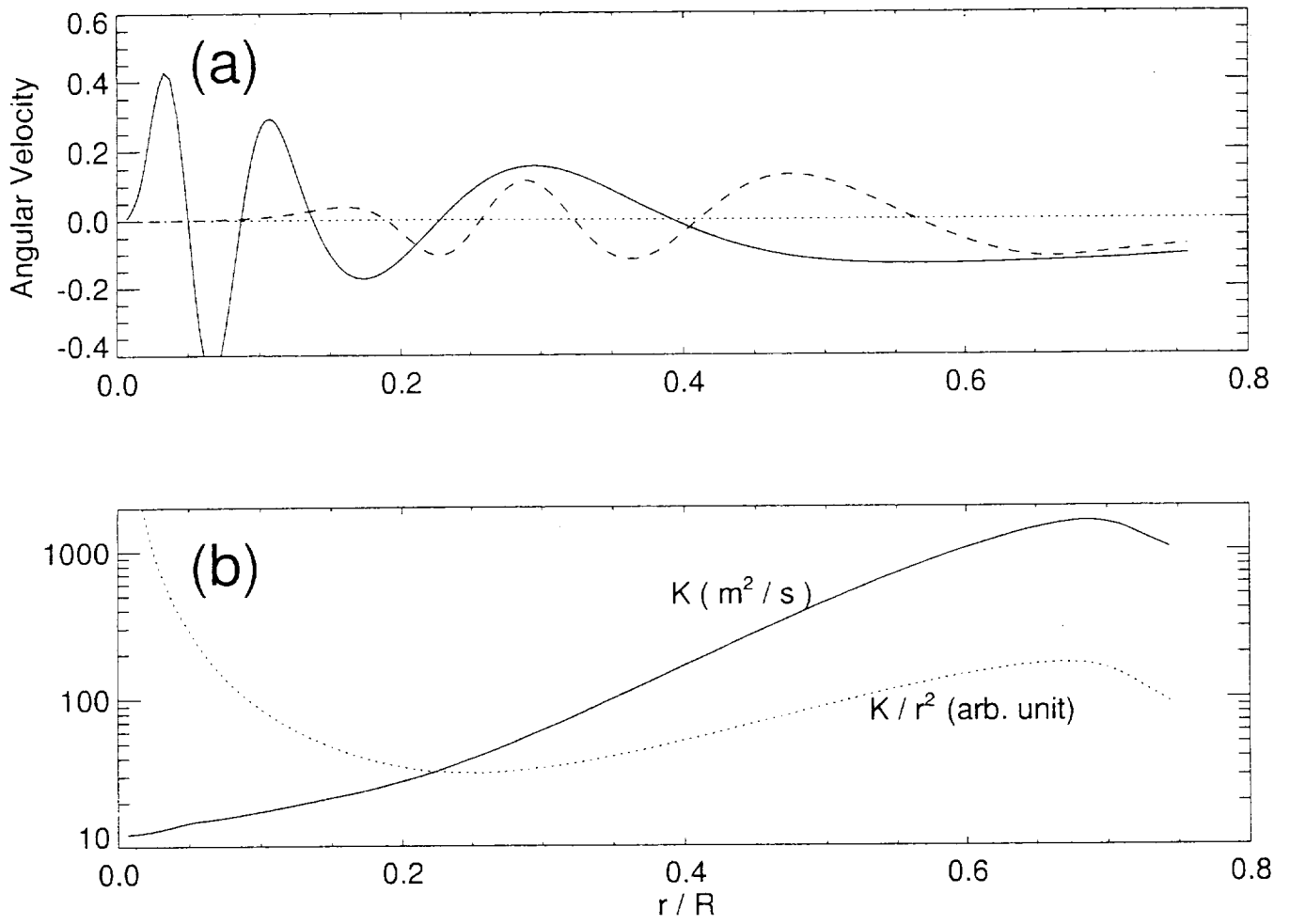


Fig. 6 a) The radial dependence of angular velocity for the g-mode  $(L,n) = (5, 5)$  in a SSM (solid) and in the proposed model (dashed). b) The radiative diffusivity  $K$  strongly damps g-modes below  $0.1 R$  (dotted curve, see text). Modes in the proposed model largely avoid this damping.



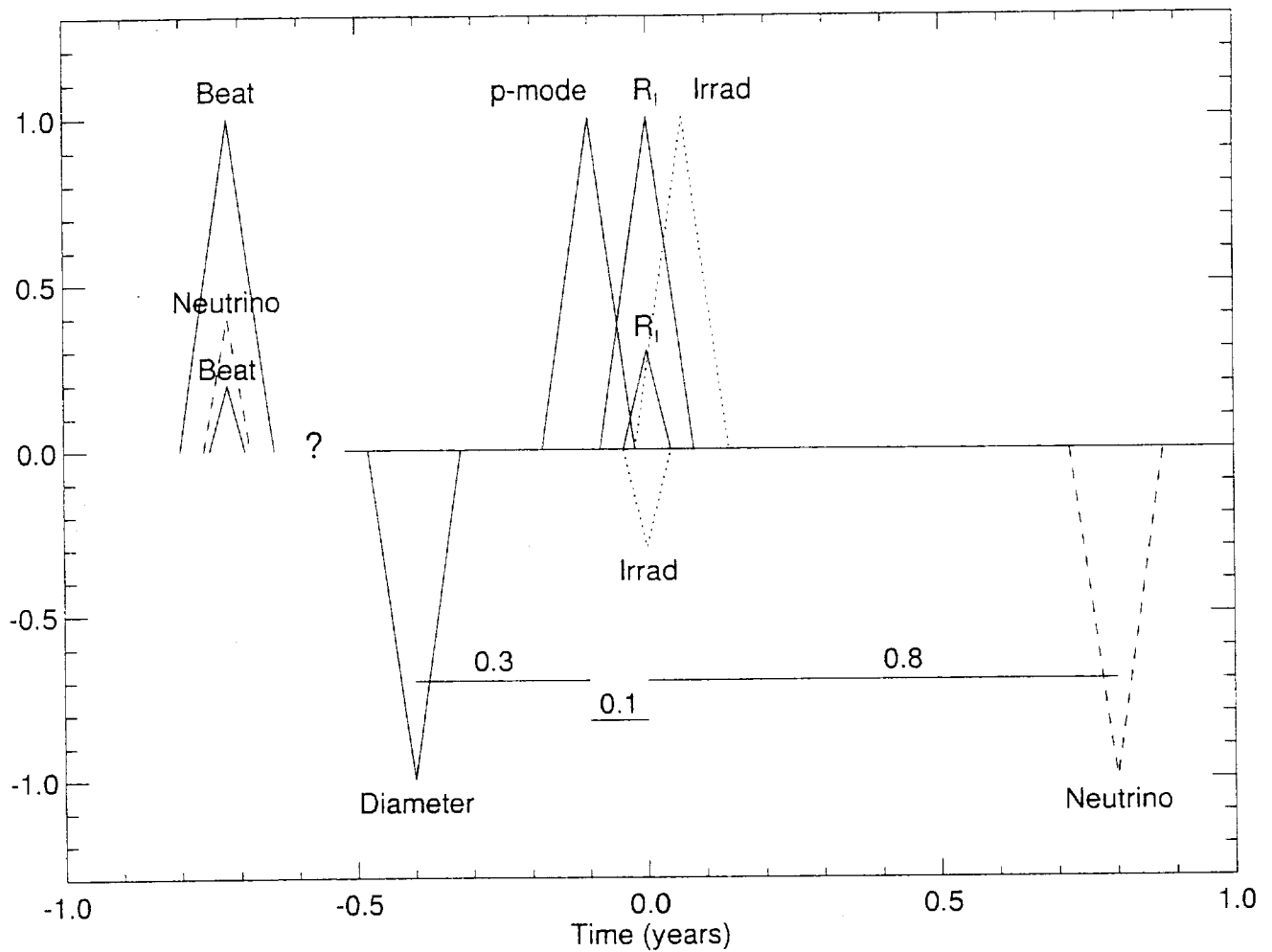


Fig. 7 Sequence of events following a beat between standing g-modes. Extremes of decadal trends are drawn with unit amplitude while monthly fluctuations are drawn smaller. For example, the sunspot number  $R_I$  peaks 0.8 years before the neutrino flux reaches a minimum. No observations yet establish the lead time for the beats at the left but theory suggests they are probably simultaneous with the minimum of solar diameter (see text).

